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T-Odd Triple-Product Correlations in Hadronic b Decays

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Abstract

We study T-violating triple-product asymmetries in the quark-level decay $b \to su\bar{u}$ within the standard model (SM). We find that only two types of triple products are non-negligible. The asymmetry in $\vec{p}_u \cdot (\vec{s}_u \times \vec{s}_{\bar{u}})$ or $\vec{p}_{\bar{u}} \cdot (\vec{s}_u \times \vec{s}_{\bar{u}})$ can be as large as about 5%. It can be probed in $B \to V_1 V_2$ decays, where V_1 and V_2 are vector mesons. And the asymmetry in $\vec{s}_b \cdot (\vec{p}_u \times \vec{p}_s)$ can be in the range 1%–3%. One can search for this signal in the decay $\Lambda_b \to \Lambda \pi^+ \pi^-$ or in $B^* \to X_s X$, where X_s and X then each decay into two mesons. All other triple-product asymmetries are expected to be small within the SM. This gives us new methods of searching for new physics.

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1 Introduction

These are exciting times for B physics. The CDF collaboration has made a measurement of the CP-violating phase $\sin 2\beta = 0.79^{+0.41}_{-0.44}$ [1], leading to the nontrivial constraint $\sin 2\beta > 0$ at 93% C.L. The asymmetric e^+e^- B-factories BaBar and Belle are now running and will hopefully make measurements of CP-violating rate asymmetries in the B system before too long. And in the near future, data from HERA-B and hadron colliders will add to our knowledge of CP violation in the B system.

The purpose of all this activity is to test the standard model (SM) explanation of CP violation. In the SM, CP violation, which to date has been only seen in the kaon system, is due to the presence of a nonzero complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix V. In this scenario, one expects large CP-violating effects in B decays, and the above experiments are searching for such signals.

The CP-violating signals which have been the most extensively studied are rate asymmetries in B decays [2]. Measurements of such asymmetries will allow one to cleanly probe the interior angles α, β and γ of the so-called unitarity triangle [3], which will in turn provide important tests of the SM.

However, there is another class of CP-violating signals which has received relatively little attention: triple-product correlations [4]. In a given decay, it may be possible to measure the momenta and/or spins of the particles involved. From these one can construct triple products of the form $\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3)$, where each v_i is a spin or momentum. Such triple products are odd under time reversal (T) and hence, by the CPT theorem, are also potential signals of CP violation. (Note that there is a technical distinction to be made here: although the action of T changes the sign of a triple product, if a triple product changes sign, it is not necessarily due to the T transformation. This is because, in addition to reversing spins and momenta, the time reversal symmetry T also exchanges the initial and final states. Thus, in a particular decay, a nonzero triple product is not necessarily a signal of T (and CP) violation. For this reason, in what follows we refer to triple-product asymmetries as *T-odd* effects. We also show how to establish the presence of a true signal of T violation.)

To establish the presence of a nonzero triple-product correlation, one constructs a T-odd asymmetry of the form

$$A_T \equiv \frac{\Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) > 0) - \Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) < 0)}{\Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) > 0) + \Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) < 0)} , \tag{1}$$

where Γ is the decay rate for the process in question. Unfortunately, triple-product correlations suffer from a well-known complication: their signals can be faked by the presence of strong phases, even if there is no CP violation. (As noted above, this is because such correlations are not true T-violating signals.) That is, one typically finds that

$$A_T \propto \sin(\phi + \delta), \tag{2}$$

where ϕ is a weak, CP-violating phase and δ is a strong phase. From this we see that if $\delta \neq 0$, a triple-product correlation will appear, even in the absence of CP violation (i.e. if $\phi = 0$).

To remedy this, one can construct the *T-violating asymmetry*:

$$\mathcal{A}_T \equiv \frac{1}{2}(A_T - \bar{A}_T) , \tag{3}$$

where $\bar{A}_T$ is the T-odd asymmetry measured in the CP-conjugate decay process. This is a true T-violating signal in that it is nonzero only if $\phi \neq 0$ (i.e. if CP violation is present). Furthermore, unlike decay-rate asymmetries in direct CP violation, a nonzero $\mathcal{A}_T$ does not require the presence of a nonzero strong phase. Indeed:

$$\mathcal{A}_T \propto \sin\phi \cos\delta , \tag{4}$$

so that the signal is maximized when the strong phase is zero.

As with all CP-violating signals, (at least) two decay amplitudes are necessary to produce a triple-product correlation. Such correlations have been studied in semileptonic B decays [5]. However, since there is only a single amplitude in the SM, any such signal can occur only in the presence of new physics.

To our knowledge, the only study of triple products in the SM has been made by Valencia [6], who examined the decay $B \to V_1 V_2$, where V_1 and V_2 are vector mesons. He looked at triple products of the form $\vec{k} \cdot (\vec{\epsilon}_1 \times \vec{\epsilon}_2)$, where $\vec{\epsilon}_1$ and $\vec{\epsilon}_2$ are the polarizations of V_1 and V_2, respectively, and $\vec{k}$ is the momentum of one of the vector mesons. Since the calculation was done at the meson level, estimates of the various form factors were needed. The conclusion of this study was that, within the SM, one could expect a T-violating asymmetry at the level of several percent.

In this paper we re-examine the question of triple products in the SM using a complementary approach. In particular, we search for triple-product correlations at the quark level. The motivation is the following: if a significant triple-product correlation exists at the hadron level, it must also exist at the quark level. After all, given that QCD (which is responsible for hadronization) is CP-conserving, it is difficult to see how one can generate a large T-violating asymmetry at the hadron level if it is absent at the quark level.

Of course, the converse is not necessarily true: a large T-violating effect at the quark level might be "washed out" during hadronization, since the spins and momenta of the quarks may not correlate well with the spins and momenta of the hadrons. (The most obvious example of this is if spin-0 mesons are involved. In this case no information about the spins of the constituent quarks can be obtained.) Thus, there may be considerable hadronic uncertainty in taking a nonzero quark-level signal and applying it at the hadron level.

With this in mind, in this paper we examine the inclusive decay $b \to s u \bar{u}$ within the SM. If there is a large $\vec{k} \cdot (\vec{\epsilon}_1 \times \vec{\epsilon}_2)$ triple product in $B \to V_1 V_2$, there should also be a large triple product at the quark level of the form $\vec{p} \cdot (\vec{s} \times \vec{s}')$, where $\vec{p}$ is the momentum of one of the quarks, and $\vec{s}$ and $\vec{s}'$ are the spins of two of the light quarks. And indeed, we find that the quark-level T-violating asymmetry due to the triple product $\vec{p}_u \cdot (\vec{s}_u \times \vec{s}_{\bar{u}})$ or $\vec{p}_{\bar{u}} \cdot (\vec{s}_u \times \vec{s}_{\bar{u}})$ can be as large as about 5%. This strongly supports Valencia's conclusion that the SM predicts a measurable T-violating asymmetry in $B \to V_1 V_2$.

However, we also find another significant T-violating signal in $b \to s u \bar{u}$. It is due to the triple-product $\vec{s}_b \cdot (\vec{p}_u \times \vec{p}_s)$, which involves the b-quark spin and the momenta of the s and u quarks. In the SM, this signal turns out to be in the range of 1% to 3% of the total rate, which may be measurable. It might be observable in decays such as $\Lambda_b \to \Lambda \pi^+ \pi^-$ (though the usual caveats about large hadronic uncertainties apply).

Finally, it is also important to note which T-violating signals are *not* present. For example, we find that there are no significant T-violating asymmetries in the SM which involve the spin of the s-quark. Thus, should such an asymmetry be measured, it would be a clear sign of new physics.

In Sec. 2, we compute the triple products present in the decay $b \to s u \bar{u}$, and estimate their sizes. We discuss possible hadron-level applications in Sec. 3. We conclude in Sec. 4.

2 Triple Products in $b \to s u \bar{u}$

In the inclusive decay $b \to s u \bar{u}$, the amplitude has two dominant contributions: the tree diagram (T) due to W-boson exchange and the loop-level strong penguin diagram (P). Furthermore, the penguin amplitude contains two dominant terms, P_1 and P_2 [7]. These various contributions are given by:

$$
\begin{aligned}
T &= \frac{4G_F}{\sqrt{2}} V_{ub} V_{us}^* \left[\bar{u}\gamma_\mu \gamma_L b\right] \left[\bar{s}\gamma^\mu \gamma_L v_u\right] e^{i\delta_t} \ , \\
P_1 &= -\frac{\alpha_s G_F}{\sqrt{2}\pi} F_1^c V_{cb} V_{cs}^* \left[\bar{s} t^\alpha \gamma_\mu \gamma_L b\right] \left[\bar{u} t_\alpha \gamma^\mu v_u\right] e^{i\delta_1} \ , \\
P_2 &= -\frac{\alpha_s G_F}{\sqrt{2}\pi} \left[\frac{-i m_b}{q^2} F_2\right] V_{tb} V_{ts}^* \left[\bar{s} t^\alpha \sigma_{\mu\nu} q^\nu \gamma_R b\right] \left[\bar{u} t_\alpha \gamma^\mu v_u\right] e^{i\delta_2} \ .
\end{aligned} \tag{5}
$$

In the above, $\gamma_{L(R)} = (1 \mp \gamma_5)/2$, the t^α are the Gell-Mann matrices, and the δ_i are the strong phases. In P_2, q is the momentum of the internal gluon. The factors F_1^c and F_2 are functions of (m_c^2/M_W^2) and (m_t^2/M_W^2), respectively, and take the values $F_1^c \simeq 5.0$ and $F_2 \simeq 0.2$ for $m_t = 160$ GeV [7]. P_1 and P_2 are often called the *chromoelectric dipole moment* term and *chromomagnetic dipole moment* term, respectively.

The next step is the calculation of the square of the decay amplitude. We have:

$$
|\mathcal{M}|^2 = |T|^2 + |P_1|^2 + |P_2|^2 + 2Re\left(T^\dagger P_1\right) + 2Re\left(T^\dagger P_2\right) + 2Re\left(P_1^\dagger P_2\right) \ . \tag{6}
$$

We will see below that the dominant term here is $|P_1|^2$.

We find triple products in all of the interference terms above (i.e. the last three terms of $|\mathcal{M}|^2$). Before giving the specific forms of these triple products, we make the following general remarks:

- In the calculation, we neglect the masses of the light quarks s, u and $\bar{u}$, but we keep the spins (i.e. polarization four-vectors) of these particles (at least to begin with). It turns out that there are no triple products involving the polarization of the s quark. (In other words, such terms are suppressed by at least m_s/m_b.) In light of this, in our results below, we automatically sum over the s-quark spin states.

  This is an interesting result: it suggests that if a triple product involving the s-quark polarization is observed experimentally, it is probably due to physics beyond the SM.

- Since the B meson has spin 0, triple products in $B \to V_1V_2$ cannot involve the spin of the b-quark. If one sums over the spin of the b-quark, the only term which contains triple products is the $T - P_1$ interference term. We will therefore use this term to estimate the size of the T-violating asymmetry in $B \to V_1V_2$ [6].

- If the spins of the u and $\bar{u}$ quarks cannot be measured, one can then take them to be unpolarized, i.e. we sum over their polarizations. In this case, only the $T - P_2$ and $P_1 - P_2$ interferences contain a triple product. This unique signal takes the form $\vec{s}_b \cdot (\vec{p}_u \times \vec{p}_s)$.

- In all interference terms, there are triple products which involve the three polarizations $\vec{s}_b$, $\vec{s}_u$ and $\vec{s}_{\bar{u}}$. Experimentally, such signals will be extremely difficult to measure, and so are of less interest than the others described here.

2.1 $T - P_1$ interference

Keeping explicit the spins of the b-, u- and $\bar{u}$-quarks, the T-odd piece of the $T - P_1$ interference term is

$$\begin{aligned}
\left[\sum_{s\ spins} 2Re\left(T^\dagger P_1\right)\right]_{T-odd} &= \frac{16\alpha_s G_F^2 F_1^c}{3\pi}\, Im\Big[V_{cs}^* V_{cb} V_{us} V_{ub}^* e^{i(\delta_1-\delta_t)}\Big] \\
&\times \Big\{ 2(p_b \cdot s_u)\,\epsilon_{\mu\nu\rho\xi}\, p_b^\mu p_u^\nu p_{\bar{u}}^\rho s_{\bar{u}}^\xi - 2(p_b \cdot p_u)\,\epsilon_{\mu\nu\rho\xi}\, p_b^\mu s_u^\nu p_{\bar{u}}^\rho s_{\bar{u}}^\xi + m_b^2\,\epsilon_{\mu\nu\rho\xi}\, p_u^\mu s_u^\nu p_{\bar{u}}^\rho s_{\bar{u}}^\xi \\
&\quad + m_b \Big[(s_b \cdot s_u)\,\epsilon_{\mu\nu\rho\xi}\, p_s^\mu p_u^\nu p_{\bar{u}}^\rho s_{\bar{u}}^\xi - (s_b \cdot p_u)\,\epsilon_{\mu\nu\rho\xi}\, p_s^\mu p_{\bar{u}}^\nu s_{\bar{u}}^\rho s_u^\xi \\
&\qquad - (p_s \cdot p_{\bar{u}})\,\epsilon_{\mu\nu\rho\xi}\, p_u^\mu s_b^\nu s_u^\rho s_{\bar{u}}^\xi + (p_s \cdot s_{\bar{u}})\,\epsilon_{\mu\nu\rho\xi}\, p_u^\mu p_{\bar{u}}^\nu s_b^\rho s_u^\xi \Big] \Big\} .
\end{aligned} \tag{7}$$

Here, p_i is the 4-momentum of the i-quark and s_i is its polarization four-vector. Triple products[3] are found in the terms $\epsilon_{\mu\nu\rho\xi} v_1^\mu v_2^\nu v_3^\rho v_4^\xi$.

In the above expression, we see that there are two categories of triple products: those which involve s_b, the b-quark polarization, and those which do not. Those terms which include s_b (the last four terms in Eq. 7) also include the polarizations of the u- and $\bar{u}$-quarks (s_u and $s_{\bar{u}}$). Since all three spins must be measured, these triple products will be extremely difficult to observe experimentally. Because of this, it is the first three terms of Eq. 7 which most interest us, and we therefore isolate them by averaging over s_b.

[3]Note that, due to the identity $g_{\alpha\beta}\,\epsilon_{\mu\nu\rho\xi} - g_{\alpha\mu}\,\epsilon_{\beta\nu\rho\xi} - g_{\alpha\nu}\,\epsilon_{\mu\beta\rho\xi} - g_{\alpha\rho}\,\epsilon_{\mu\nu\beta\xi} - g_{\alpha\xi}\,\epsilon_{\mu\nu\rho\beta} = 0$, not all terms of the form $v_1 \cdot v_2\,\epsilon_{\mu\nu\rho\xi} v_3^\mu v_4^\nu v_5^\rho v_6^\xi$ are necessarily independent.

Of course, as written, the terms $\epsilon_{\mu\nu\rho\xi} v_1^\mu v_2^\nu v_3^\rho v_4^\xi$ involve only four-vectors, and therefore do not look like triple products. In order to identify the triple products implicit in these terms, we have to choose a particular frame of reference. The most natural choice is the rest frame of the b-quark, in which case Eq. 7 then takes the form

$$\left[\frac{1}{2} \sum_{b,s\ spins} 2Re\left(T^\dagger P_1\right) \right]_{T-odd} = \frac{16\alpha_s G_F^2 F_1^c}{3\pi} Im\Big[V_{cs}^* V_{cb} V_{us} V_{ub}^* e^{i\Delta_{1t}} \Big] m_b^2$$
$$\times \left\{ s_u^0 \, \vec{p}_u \cdot (\vec{p}_{\bar{u}} \times \vec{s}_{\bar{u}}) + E_u \, \vec{p}_{\bar{u}} \cdot (\vec{s}_u \times \vec{s}_{\bar{u}}) + s_{\bar{u}}^0 \, \vec{p}_u \cdot (\vec{p}_{\bar{u}} \times \vec{s}_u) + E_{\bar{u}} \, \vec{p}_u \cdot (\vec{s}_u \times \vec{s}_{\bar{u}}) \right\} , \quad (8)$$

where $\Delta_{1t} \equiv \delta_1 - \delta_t$. We therefore see that there are, in fact, four distinct triple products in the $T - P_1$ interference term.

These triple products depend on the polarization four-vectors of the u- and $\bar{u}$-quarks, whose most general form is [8]

$$s_i^\mu = \left(\frac{\vec{n}_i \cdot \vec{p}_i}{m_i} \ , \ \vec{n}_i + \frac{\vec{n}_i \cdot \vec{p}_i}{m_i(E_i + m_i)} \vec{p}_i \right) , \quad (9)$$

for $i = u, \bar{u}$. In the above, $\vec{n}_i$ is the polarization vector of the i-quark in its rest frame, and satisfies $|\vec{n}_i| = 1$.

The triple products in Eq. (8) all involve two spins, which makes their evaluation somewhat problematic. Since one has to choose directions for $\vec{s}_u$ and $\vec{s}_{\bar{u}}$, there are an infinite number of possibilities. We have studied three cases:

1. The polarizations of the u- and $\bar{u}$-quarks in their respective centre-of-mass frames both point in a particular direction, say $\vec{n}_u = \vec{n}_{\bar{u}} = \hat{z}$.
2. The polarizations of the u- and $\bar{u}$-quarks are perpendicular, say $\vec{n}_u = \hat{z}$ and $\vec{n}_{\bar{u}} = \hat{x}$.
3. The polarization of the u-quark is longitudinal, $\vec{n}_u = \hat{p}_u$ while $\vec{n}_{\bar{u}} = \hat{z}$.

In the following we will refer to these three scenarios as Case I, Case II and Case III, respectively.

In order to compute the size of these triple-product asymmetries, in addition to integrating over phase space, we also need estimates of the sizes of the weak and strong phases. In the Wolfenstein parametrization [9], we can write the T-odd combination of CKM and strong phases as

$$Im\Big[V_{cs}^* V_{cb} V_{us} V_{ub}^* \, e^{i\Delta_{1t}} \Big] = A^2 \lambda^6 \left[\eta \cos\Delta_{1t} + \rho \sin\Delta_{1t} \right] . \quad (10)$$

CP violation in the CKM matrix is parametrized by the parameter η. As discussed in the introduction, nonzero strong phases can fake a T-violating signal. The term $\rho \sin \Delta_{1t}$ in the above expression is an example of such a fake signal. However, by forming a true T-violating asymmetry $\mathcal{A}_T$ (Eq. 3), one can eliminate this fake signal. In this case $\mathcal{A}_T \propto \eta \cos \Delta_{1t}$.

At present, η is constrained to lie in the range $0.23 \le \eta \le 0.50$ [10]. Turning to the strong phase, the tree-level phase δ_t is usually assumed to be small: the logic is that, roughly speaking, the quarks will hadronize before having time to exchange gluons. On the other hand, for the $b \to s u \bar{u}$ penguin amplitude, it is often assumed that strong phases come from the absorptive part of the penguin contribution [11]. Since P_1 involves an internal c-quark, it is possible that $\delta_1 \neq 0$, which of course implies that $\Delta_{1t} \neq 0$. Even so, for simplicity, in our calculation we assume that Δ_{1t} is small enough that $\cos \Delta_{1t} \simeq 1$ is a good approximation. However, the reader should be aware that the asymmetries may be reduced should this strong phase be large. (Note that the T-violating signal is maximal when $\cos \Delta_{1t} = 1$. For comparison, direct CP-violating rate asymmetries require the strong phase to be nonzero.)

We have performed the phase-space integration using the computer program RAMBO. For each of the three cases above we have calculated the four T-violating asymmetries [see Eq. (3)] $\mathcal{A}_T^1$, $\mathcal{A}_T^2$, $\mathcal{A}_T^3$, and $\mathcal{A}_T^4$, which correspond respectively to the four triple products of Eq. (8): $s_u^0\, \vec{p}_u \cdot (\vec{p}_{\bar{u}} \times \vec{s}_{\bar{u}})$, $E_u\, \vec{p}_{\bar{u}} \cdot (\vec{s}_u \times \vec{s}_{\bar{u}})$, $s_{\bar{u}}^0\, \vec{p}_u \cdot (\vec{p}_{\bar{u}} \times \vec{s}_u)$, and $E_{\bar{u}}\, \vec{p}_u \cdot (\vec{s}_u \times \vec{s}_{\bar{u}})$. We have taken $\eta = 0.4$. Our results are as follows.

1. Case I: The asymmetries $\mathcal{A}_T^1$ and $\mathcal{A}_T^3$ are both negligible. However, we find that $\mathcal{A}_T^2 = \mathcal{A}_T^4 \simeq 4.6\%$.
2. Case II: $\mathcal{A}_T^1$ and $\mathcal{A}_T^3$ are negligible, while $\mathcal{A}_T^2 = \mathcal{A}_T^4 \simeq 3.9\%$.
3. Case III: Here the triple products $\vec{p}_u \cdot (\vec{p}_{\bar{u}} \times \vec{s}_u)$ and $\vec{p}_u \cdot (\vec{s}_u \times \vec{s}_{\bar{u}})$ vanish identically, so that $\mathcal{A}_T^3 = \mathcal{A}_T^4 = 0$. We find that the other two asymmetries are tiny: $\mathcal{A}_T^2 = -\mathcal{A}_T^1 \simeq 0.09\%$.

From the above, we conclude that the asymmetries $\mathcal{A}_T^1$ and $\mathcal{A}_T^3$ are both very small in the SM, and that $\mathcal{A}_T^2$ and $\mathcal{A}_T^4$ can be as large as about 5%.

Note that the triple product in $B \to V_1 V_2$ discussed by Valencia [6] is of the form $\vec{k} \cdot (\vec{\epsilon}_1 \times \vec{\epsilon}_2)$. In Eq. (8), it is the terms $\vec{p}_{\bar{u}} \cdot (\vec{s}_u \times \vec{s}_{\bar{u}})$ and $\vec{p}_u \cdot (\vec{s}_u \times \vec{s}_{\bar{u}})$ which could potentially give such a triple-product signal. We have found that the asymmetries $\mathcal{A}_T^2$ and $\mathcal{A}_T^4$, which correspond to these triple products, can be reasonably big ($\lesssim 5\%$). This is consistent with the results found by Valencia at the meson level, and suggests that the SM does indeed predict a measurable T-violating asymmetry in $B \to V_1 V_2$ decays. (Of course, due to our difficulties in understanding hadronization, it is not possible to use the quark-level result to predict the size of the asymmetry for a specific meson-level decay.)

Finally, for comparison, consider the decay-rate asymmetry, calculated by Hou for the same process [12]:

$$a_{CP}(b \to s u \bar{u}) \simeq 1.4\% \tag{11}$$

We therefore see that one expects T-violating triple-product asymmetries in $b \to s u \bar{u}$ to be considerably larger than the decay rate asymmetry.

2.2 $P_1 - P_2$ interference

The T-odd piece of the $P_1 - P_2$ interference term is

$$\begin{aligned}
\left[\sum_{s\ spins} 2Re\left(P_1^\dagger P_2\right) \right]_{T-odd} &= \frac{4\alpha_s^2 G_F^2 F_1^c F_2 m_b}{3\pi^2 q^2}\, Im\Big[V_{ts}^* V_{tb} V_{cs} V_{cb}^*\, e^{i(\delta_2 - \delta_1)} \Big] \\
&\times \Bigg\{ \Big[p_b \cdot (p_u - p_{\bar{u}})\,(1 - s_u \cdot s_{\bar{u}}) \;-\; (s_{\bar{u}} \cdot p_s)(s_u \cdot p_{\bar{u}}) +\; (s_u \cdot p_s)(s_{\bar{u}} \cdot p_u) \Big]\, \epsilon_{\mu\nu\rho\xi}\, p_b^\mu s_b^\nu p_u^\rho p_s^\xi \\
&\quad + \Big[(s_u \cdot p_{\bar{u}})(p_u \cdot p_s) - \frac{q^2}{2}(s_u \cdot p_s) \Big]\, \epsilon_{\mu\nu\rho\xi}\, p_b^\mu s_b^\nu p_s^\rho s_{\bar{u}}^\xi \\
&\quad + \Big[(s_{\bar{u}} \cdot p_u)(p_{\bar{u}} \cdot p_s) - \frac{q^2}{2}(s_{\bar{u}} \cdot p_s) \Big]\, \epsilon_{\mu\nu\rho\xi}\, p_b^\mu s_b^\nu p_s^\rho s_u^\xi \Bigg\}\ .
\end{aligned} \tag{12}$$

Here, if we average over the b-quark spin states, there is no T-violating signal at all.

We note that most of the terms in Eq. 12 correspond to triple products in which three spins must be measured. As we have already discussed, such signals are very difficult to observe experimentally, and so do not interest us. There is one term, however, which does not involve three spins, and it can be isolated by summing over the u- and $\bar{u}$-quark spin states:

$$\begin{aligned}
\left[\sum_{u,\bar{u},s\ spins} 2Re\left(P_1^\dagger P_2\right) \right]_{T-odd} &= \frac{16\alpha_s^2 G_F^2 F_1^c F_2 m_b}{3\pi^2 q^2}\, Im\Big[V_{ts}^* V_{tb} V_{cs} V_{cb}^*\, e^{i(\delta_2 - \delta_1)} \Big] \\
&\quad \times p_b \cdot (p_u - p_{\bar{u}})\ \epsilon_{\mu\nu\rho\xi} p_b^\mu s_b^\nu p_u^\rho p_s^\xi\ .
\end{aligned} \tag{13}$$

In the rest frame of the b-quark, the triple product takes the form $m_b^2 (E_u - E_{\bar{u}})\, \vec{s}_b \cdot (\vec{p}_u \times \vec{p}_s)$. Integrating over phase space with RAMBO, we find that the $P_1 - P_2$ T-violating asymmetry is $O(10^{-5})$, which is negligible.

2.3 $T - P_2$ interference

Like $P_1 - P_2$ interference, the $T - P_2$ interference term contains two types of triple produts: (i) those involving a single quark polarization, s_b, and (ii) those involving the three polarization four-vectors s_b, s_u and $s_{\bar{u}}$. As usual, we are not interested in triple products involving three spins, and so we can therefore sum over s_u and $s_{\bar{u}}$. The T-odd piece of the $T - P_2$ interference term is then given by

$$\left[\sum_{u,\bar{u},s \ spins} 2Re\left(T^\dagger P_2\right) \right]_{T-odd} = \frac{128\alpha_s G_F^2 F_2 m_b}{3\pi q^2} Im\Big[V_{ts}^* V_{tb} V_{us} V_{ub}^* \, e^{i\Delta_{2t}}\Big] \times p_s \cdot p_u \, \epsilon_{\mu\nu\rho\xi} \, p_b^\mu s_b^\nu p_u^\rho p_s^\xi \ , \tag{14}$$

where $\Delta_{2t} \equiv \delta_2 - \delta_t$.

As was the case for the $T - P_1$ interference term, the T-violating asymmetry $\mathcal{A}_T$ is proportional to $\eta \cos\Delta_{2t}$. And, as before, we expect the δ_t piece of the strong phase Δ_{2t} to be small. However, there is a difference here compared to the $T - P_1$ case: previously, the penguin amplitude P_1 involved an internal c-quark, and so it was possible that the strong phase δ_{1t}, which is related to the absorptive part of the amplitude, could be sizeable. Here, the triple product involves only the t-quark penguin contribution P_2, which is purely dispersive, and so leads to $\delta_2 = 0$. Thus, it is an excellent approximation to set $\Delta_{2t} \simeq 0$.

In the rest frame of the b-quark, the triple-product of Eq. 14 is $m_b \, p_s \cdot p_u \, \vec{s}_b \cdot (\vec{p}_u \times \vec{p}_s)$. Integrating over phase space using RAMBO, and using the allowed range for η, we find that this T-violating triple-product asymmetry can be of the order of several percent:

$$1.3\% \lesssim \mathcal{A}_T(b \to s u \bar{u}) \lesssim 3.2\% \ . \tag{15}$$

This could conceivably be measured at a future experiment.

Furthermore, if it is found that this asymmetry is considerably larger than the above values, it is probably a signal of new physics. For example, in some models of new physics, the chromomagnetic dipole moment F_2 can be enhanced up to ten times its SM value [13]. This will clearly have an enormous affect on the above asymmetry.

3 Applications

In the previous section, in our study of the quark-level decay $b \to s u \bar{u}$ within the SM, we found two classes of triple products whose T-violating asymmetry is large. They are: (i) $E_u\, \vec{p}_{\bar{u}} \cdot (\vec{s}_u \times \vec{s}_{\bar{u}})$ and $E_{\bar{u}}\, \vec{p}_u \cdot (\vec{s}_u \times \vec{s}_{\bar{u}})$, and (ii) $m_b\, p_s \cdot p_u\, \vec{s}_b \cdot (\vec{p}_u \times \vec{p}_s)$. The next obvious question is then: how can one test these results?

The ideal way would be to make triple-product measurements *inclusively*. If this were possible, then it would be straightforward to compare the experimental values with the theoretical predictions. However, this may not be experimentally feasible, in which case we must turn to exclusive B decays.

The first class of triple-product asymmetries can be studied in $B \to V_1 V_2$ decays which are dominated by the quark-level process $b \to s u \bar{u}$. Examples of such decays include $\overline{B_d^0} \to \rho K^*$, $\overline{B_s^0} \to K^{*+} K^{*-}$, $B_c^- \to D^* K^{*-}$, etc. These have been examined by Valencia, and we refer the reader to Ref. [6] for details.

Turning to the second class of triple products, it is clear that we cannot use decays of B mesons to obtain these asymmetries: since the B-meson spin is zero, there is no way to measure the spin of the b-quark (which is the only spin contributing to the triple product). However, one possibility would be to use the Λ_b baryon, whose spin is largely that of the b quark. For example, we can consider the process $\Lambda_b \to \Lambda \pi^+ \pi^-$. The triple product $\vec{s}_b \cdot (\vec{p}_u \times \vec{p}_s)$ can be roughly equated to $\vec{s}_{\Lambda_b} \cdot (\vec{p}_{\pi^+} \times \vec{p}_{\Lambda})$.

Another possibility is to consider a B^* meson decaying into any two mesons $X_s X$, where X_s and X then decay respectively into mesons $\Phi_1 \Phi_2$ and $\Phi_3 \Phi_4$ (since with only one spin, we need three independent momenta). The triple product $\vec{s}_b \cdot (\vec{p}_u \times \vec{p}_s)$ may then be roughly related to $\vec{s}_{B^*} \cdot (\vec{p}_{\Phi_1} \times \vec{p}_{\Phi_3})$.

In the above, we have been deliberately vague about the relationship between the triple products at the quark and hadron levels. We do not understand hadronization of quarks into mesons all that well, and when one adds the complication of relating the quark spins to the hadron spins, things are even more uncertain. Given that there is a large quark-level triple-product asymmetry, there may be one at the hadron level. However, if experiment does not find such an asymmetry, this may not be a sign of new physics – it may simply mean that the asymmetry has been washed out during hadronization. Regardless of the results, studies of this kind are likely to help us understand how quarks hadronize into mesons and baryons.

Finally, we note that certain quark-level triple products are predicted to be small in the SM. For example, triple products involving the spin of the s-quark are suppressed by powers of its mass. Hence, if a T-violating asymmetry due to a triple product involving the s-quark spin were found to be sizeable, this would probably indicate the presence of new physics. The decay $\Lambda_b \to \Lambda \pi^+ \pi^-$, which was mentioned above, can be used to test this. The spin of the Λ is due mostly to the s-quark spin. So any T-violating asymmetry involving the spin of the Λ, such as $\vec{s}_{\Lambda_b} \cdot (\vec{s}_\Lambda \times \vec{p}_\Lambda)$, should be tiny in the SM.

As another example, recall that we found that $T - P_1$ interference produced the triple products $s_u^0 \, \vec{p}_u \cdot (\vec{p}_{\bar{u}} \times \vec{s}_{\bar{u}})$ and $s_{\bar{u}}^0 \, \vec{p}_u \cdot (\vec{p}_{\bar{u}} \times \vec{s}_u)$. However, the corresponding T-violating asymmetries $\mathcal{A}_T^1$ and $\mathcal{A}_T^3$ turned out to be suppressed dynamically. Consider the decay of a B-meson to two vector mesons, $B \to V_1 V_2$, where the V_2 then subsequently decays to two mesons $\Phi_1 \Phi_2$. Roughly speaking, one can relate $s_u^0 \, \vec{p}_u \cdot (\vec{p}_{\bar{u}} \times \vec{s}_{\bar{u}})$ to $\epsilon_{V_1}^0 \, \vec{p}_{V_1} \cdot (\vec{\epsilon}_{V_2} \times \vec{p}_{\Phi_1})$. Thus, the measurement of a nonzero value for this latter triple-product asymmetry would be a signal for new physics.

4 Conclusions

We have calculated the quark-level triple-product correlations in the decay $b \to s u \bar{u}$ within the standard model. Although several such triple products are present, we find that only two types lead to sizeable T-violating asymmetries.

The first type includes $\vec{p}_u \cdot (\vec{s}_u \times \vec{s}_{\bar{u}})$ and $\vec{p}_{\bar{u}} \cdot (\vec{s}_u \times \vec{s}_{\bar{u}})$. We find that the corresponding T-violating asymmetries can be as large as about 5%. This triple product can be probed in $B \to V_1 V_2$ decays, where V_1 and V_2 are vector mesons [6].

The second type is $\vec{s}_b \cdot (\vec{p}_u \times \vec{p}_s)$, where $\vec{s}_b$ is the polarization of the b-quark, and $\vec{p}_u$ and $\vec{p}_s$ are the momenta of the u- and s-quark, respectively. We calculate that the T-violating asymmetry for this triple product is in the range 1%–3%, which may be measurable. There are several ways to try to search for this triple-product asymmetry. For example, one could study the decay $\Lambda_b \to \Lambda \pi^+ \pi^-$, looking for a nonzero triple product $\vec{s}_{\Lambda_b} \cdot (\vec{p}_{\pi^+} \times \vec{p}_\Lambda)$. Another possibility is to examine the decay $B^* \to X_s X$, where X_s and X then decay respectively into mesons $\Phi_1 \Phi_2$ and $\Phi_3 \Phi_4$, and to search for the triple product $\vec{s}_{B^*} \cdot (\vec{p}_{\Phi_1} \times \vec{p}_{\Phi_3})$.

The fact that we find only two large triple-product correlations has interesting consequences. If a triple product is tiny at the quark level, it is probably tiny at the hadron level as well. After all, the hadronization of quarks into hadrons is a strong-interaction process, and QCD is CP-conserving. It is therefore difficult to see how one can generate a large triple-product correlation at the hadron level, given that it is small at the quark level. (Of course, the converse is not necessarily true: it is quite possible that a quark-level CP-violating effect might be "washed out" during hadronization.)

From the point of view of looking for physics beyond the SM, it is therefore important to identify those triple-product asymmetries which are expected to be small in the SM. If such asymmetries are found to be large, this is probably a signal of new physics. For example, we find that triple products involving the spin of the s-quark are suppressed by powers of its mass. Thus, if, for instance, a sizeable T-violating asymmetry of the form $\vec{s}_{\Lambda_b} \cdot (\vec{s}_\Lambda \times \vec{p}_\Lambda)$ were found in the decay $\Lambda_b \to \Lambda\pi^+\pi^-$, this would be compelling evidence for the presence of new physics, since the spin of the Λ is due largely to the s-quark spin.

Note that we have performed the calculation at the quark level, and the passage from quarks to hadrons is not well understood. Thus, in addition to being an interesting signal of T and CP violation, the study of T-violating triple-product asymmetries may help us understand aspects of hadronization.

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